Uva Wellassa University, Sri Lanka
End Semester Examination - December 2009
SCT 301-2 Advanced Mathematics I

Time : Two (2) hours

Answer all questions.
Calculators are allowed.
Total three (3) pages.

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01) What is meant by the principal of superposition?

(2 marks)

a) State the general form of heat equation with proper identification of terms.

(2 marks)

Solve the following heat equation,

$$\frac{\partial u}{\partial t} = \frac{\sqrt{\pi}}{2}\frac{\partial^2 u}{\partial x^2} \quad ; \quad 0 < x < \pi\,,\ t > 0$$

Given $u(0,t) = u(\pi,t) = 0$,
$u(x,0) = 2\sin\sqrt{2}\pi x + \sqrt{2}\sin 2\pi x$,
$|u(x,t)| < M$.

(10 marks)

b) State the general form of wave equation with proper identification of terms.

(2 marks)

A rope of length 2π units is stretched between two points $(-\pi, 0)$ and $(\pi, 0)$ on the x-axis. when $t = 0$ it has the shape of $x^2 - \pi^2$; $-\pi < x < \pi$. Then the rope is released from the rest. Find the displacement of the string at any time t after it is released.

(15 marks)

02) Laplace transformation ($\mathcal{L}$) is an operator. What conditions need to be satisfied if an operator to be linear.

(2 marks)

a) A mass (m) is suspended from the end of a vertical spring with spring constant κ. An external force $F(t)$ acts on the mass as well as a resistive force proportional to the instantaneous velocity. Assuming that x is the displacement of the mass at time t and that the mass starts from the rest at $t = 0$,

i. set a differential equation for the motion of m.
ii. find x at any time t.

(15 marks)

b) A resistor of $R = 30\ Ohms$, an inductor of $L = 3\ Henries$ and a battery of E Volts are connected in series with a switch S. At $t = 0$ the switch is been off, $E = 10 \sin \pi t$.
Find $I(current)$; for $t > 0$.

(8 marks)

c) Solve,

$$y''(t) - 16y(t) = 32 \quad ; \quad y(0) = 3 \ , \ y'(0) = -2$$

(10 marks)

03) What is meant by odd and even functions.

(2 marks)

a) Let $f(t)$ is a function of t, Define Laplace Transform of $f(t)$.

(1 mark)

Define gamma function and show that for any positive integer n, gamma function $= n!$

(2 marks)

i. If $f_1(t) = t^n$; $n = 1,2,3, \ldots$ show that $\mathcal{L}\{f_1(t)\} = \frac{n!}{s^{n+1}}$; $s > 0$.

ii. If $f_2(t) = t^p$; $p > -1$ show that $\mathcal{L}\{f_2(t)\} = \frac{\Gamma(p+1)}{s^{p+1}}$; $s > 0$.

iii. Explain the relationship between $\mathcal{L}\{f_1(t)\}$ and $\mathcal{L}\{f_2(t)\}$.

(9 marks)

b) Graph the following function and find its Fourier series

$$f(x) = \begin{cases} 3x & ; \quad 0 \le x \le 3 \\ 0 & ; \quad -3 < x < 0 \end{cases} \quad ; \qquad period = 6$$

(10 marks)

c) Solve the following equation, if $y(x)$ is an odd function,

$$\int_0^\infty y(x)\ \sin xt\ dx = \begin{cases} 2 & ; \quad 0 \le t < 2 \\ 3 & ; \quad 2 \le t < 3 \\ 0 & ; \quad t \ge 3 \end{cases}$$

(10 marks)