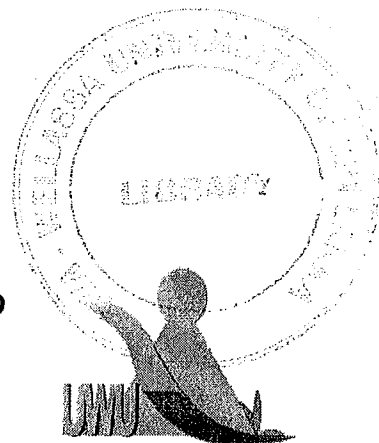


Uva Wellassa University, Sri Lanka
End Semester Examination - December 2009
SCT 301-2 Advanced Mathematics I



Time : Two (2) hours

Answer all questions.
Calculators are allowed.
Total three (3) pages.

01) What is meant by the principal of superposition?

(2 marks)

a) State the general form of heat equation with proper identification of terms.

(2 marks)

Solve the following heat equation,

$$\frac{\partial u}{\partial t} = \frac{\sqrt{\pi}}{2} \frac{\partial^2 u}{\partial x^2} \quad ; \quad 0 < x < \pi, t > 0$$

$$\begin{aligned} \text{Given } u(0, t) = u(\pi, t) = 0 \quad , \\ u(x, 0) = 2 \sin \sqrt{2}\pi x + \sqrt{2} \sin 2\pi x \quad , \\ |u(x, t)| < M. \end{aligned}$$

(10 marks)

b) State the general form of wave equation with proper identification of terms.

(2 marks)

A rope of length 2π units is stretched between two points $(-\pi, 0)$ and $(\pi, 0)$ on the x-axis. when $t = 0$ it has the shape of $x^2 - \pi^2$; $-\pi < x < \pi$. Then the rope is released from the rest. Find the displacement of the string at any time t after it is released.

(15 marks)

02) Laplace transformation (\mathcal{L}) is an operator. What conditions need to be satisfied if an operator to be linear.

(2 marks)

a) A mass (m) is suspended from the end of a vertical spring with spring constant κ . An external force $F(t)$ acts on the mass as well as a resistive force proportional to the instantaneous velocity. Assuming that x is the displacement of the mass at time t and that the mass starts from the rest at $t = 0$,

- i. set a differential equation for the motion of m .
- ii. find x at any time t .

(15 marks)

b) A resistor of $R = 30 \text{ Ohms}$, an inductor of $L = 3 \text{ Henries}$ and a battery of E Volts are connected in series with a switch S . At $t = 0$ the switch is been off,
 $E = 10 \sin \pi t$.

Find $I(\text{current})$; for $t > 0$.

(8 marks)

c) Solve,

$$y''(t) - 16y(t) = 32 \quad ; \quad y(0) = 3 \quad , \quad y'(0) = -2$$

(10 marks)



03) What is meant by odd and even functions.

(2 marks)

a) Let $f(t)$ is a function of t , Define Laplace Transform of $f(t)$.

(1 mark)

Define gamma function and show that for any positive integer n ,
gamma function = $n!$

(2 marks)

i. If $f_1(t) = t^n$; $n = 1, 2, 3, \dots$ show that $\mathcal{L}\{f_1(t)\} = \frac{n!}{s^{n+1}}$; $s > 0$.

ii. If $f_2(t) = t^p$; $p > -1$ show that $\mathcal{L}\{f_2(t)\} = \frac{\Gamma(p+1)}{s^{p+1}}$; $s > 0$.

iii. Explain the relationship between $\mathcal{L}\{f_1(t)\}$ and $\mathcal{L}\{f_2(t)\}$.

(9 marks)

b) Graph the following function and find its Fourier series

$$f(x) = \begin{cases} 3x & ; 0 \leq x \leq 3 \\ 0 & ; -3 < x < 0 \end{cases} ; \quad \text{period} = 6$$

(10 marks)

c) Solve the following equation, if $y(x)$ is an odd function,

$$\int_0^{\infty} y(x) \sin xt \, dx = \begin{cases} 2 & ; 0 \leq t < 2 \\ 3 & ; 2 \leq t < 3 \\ 0 & ; t \geq 3 \end{cases}$$

(10 marks)