

Uva Wellassa University, Sri Lanka
Btech. Science and Technology
End Semester Examination- Semester 1
January -2009



MAT 102-0 Basic Calculus

Time: Two hours

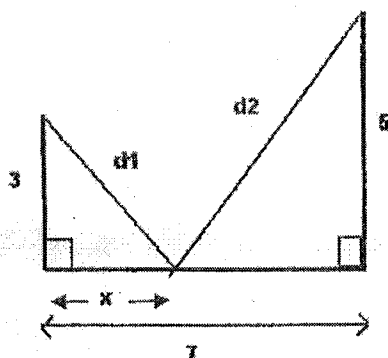
Answer Four (04) questions including Question no. (01)

No. of questions: Five (05)

Calculators are acceptable

All expressions are given in standard mathematical notations

1. a. Write the length $d1 + d2$ as a function of x .



(05 marks)

- b. A rectangle has an area equal to 100 cm^2 and a width of x . Find the perimeter of the rectangle as a function of x .

(05 marks)

- c. An approximate value in billions for the world population $p(t)$, t years after 1990 is given by

$$p(t) = \frac{10}{1 + e^{-0.03t}}$$

- i. What was the population in 1990?

(04 marks)

- ii. What will the population be in 2100?

(05 marks)

- d. Find the domain of function f defined by $f(x) = \frac{1}{\sqrt{2x-8}}$ and represent it as an inequality.

(04 marks)

- e. If function $f(x) = x^2 + 2$ and $g(x) = \ln x$, then find the function $f \circ g(x)$. (05 marks)

f. Find the following limits

i. $\lim_{x \rightarrow 0} \frac{1}{\cos x}$

ii. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$

iii. $\lim_{x \rightarrow \infty} \frac{2x^2 - x + 1}{4x^2 - 3x - 1}$

iv. $\lim_{t \rightarrow \infty} \frac{50}{2 + 3e^{-t}}$

(03 marks each)

2. a. Differentiate following functions with respect to x .

i. $f(x) = \frac{2x}{x^2 - 3}$

ii. $f(x) = 4\cos(5x - 2)$

iii. $f(x) = 2^x$

iv. $f(x) = e^{\cos x}$

(02 marks each)

b. If $f(x) = \sin x e^x$, find $f'(0)$.

(03 marks)

c. Find the first derivative of the implicit function $xy + y^2 = 4$.

(04 marks)

d. Find the third derivative of $g(x) = 12e^x + \ln(x + 2) + 2x$ with respect to x .

(05 marks)

3. a. Air is being pumped into a spherical balloon such that its radius increases at a rate of 0.75 cm/min. Find the rate of change of its volume when the radius is 5 cm.

(08 marks)

b. A book publisher is designing a book whose pages have 1 cm margins on the top and each side, and a 2 cm margin on the bottom. In this case, the publisher wants the total page area to be 150 cm². Find the page dimensions that will maximize the printed area.

(12 marks)



4. a. Evaluate following indefinite integrals.

i. $\int \left(5e^{-2x} + \frac{9}{x} \right) dx$

ii. $\int \frac{1}{\sqrt{\sin^2 x + \cos^2 x}} dx$

iii. $\int \frac{2x}{\sqrt{x^2 + 4}} dx$

iv. $\int \frac{1}{x^2 - 5x + 6} dx$

v. $\int \frac{3}{\sqrt{4-x^2}} dx$ (Hint: use the substitution $x = 2 \sin \theta$)

(04 marks each)

b. Evaluate following definite integrals.

i. $\int_0^{\pi/4} \tan x$

ii. $\int_1^2 3x^2 \cos(x^3 + 7) dx$

(2.5 marks each)

5. a. Find the area under the curve $y = x^2 + 1$ between $x = 0$ and $x = 4$ and the x -axis.

(06 marks)

b. A bacteria colony initially covered an area of 100 square millimeters. Since then the covered area has been growing at the rate of $A'(t) = \frac{180}{(t+1)^3}$ square millimeters per day.

i. Express the covered area $A(t)$ after time t as an indefinite integral.

(05 marks)

ii. Hence evaluate the area covered by bacteria after 2 days.
(Hint: $A(0) = 100 \text{ sq. mm}$)

(09 marks)