Uwa Wellassa University, Sri Lanka.

End Semester Examination - June 2009

MAT 302-2 Advanced Mathematics II

Time : Two (2) hours

Answer all questions.
Total three (3) pages.
Calculators are allowed.

1) If $f(\underline{x})$; $\underline{x} \in (x_1, x_2, x_3)$ is a function, then defined the Hessian matrix ($H(x_1, x_2, x_3)$) of $f(\underline{x})$.

(2 marks)

Let $f(\underline{x})$; $\underline{x} \in (x_1, x_2, \dots, x_n)$ is a non linear function. In Non Linear Programming (NLP) with non constraints optimization, which condition should be satisfied to have local maximum, local minimum and saddle points for $f(\underline{x})$.

(2 marks)

a) It costs a company Rs600 to produce one unit of a product. If it charges a price p and spends a dollars on advertising, it can be sold $1{,}000{,}000p^{-2}a^{1/6}$ units of the product. Find the price and advertising level that will maximize the company's profit.

(8 marks)

b) A company manufactures two products. If it charges a price p_i for product i, it can be sold q_i units of product i, where $q_1 = 60 - 3p_1 + p_2$ and $q_2 = 80 - 2p_2 + p_1$. It costs Rs2500 to produce a unit of product 1 and Rs7200 to produce a unit of product 2. How many units of each product should be produced to maximize the profit.

(8 marks)

2) State 3 methods that can be used to find the Basic Feasible Solution to a given Transportation problem, and state the best method of them. Why do you select that as the best method.

(3 marks)

a) Teleco produces TV picture tubes at three plants. Plant 1 can produce 50 tubes per week; plant 2, 100 tubes per week; and plant 3, 50 tubes per week. Tubes are shifted to three customers. The profit earned per tube depends on the site where the tube was produced and the customer who purchases the tube (see the below table). Customer 1 is willing to purchase as many as 80 tubes per week; Customer 2 , as many as 90; and customer 3, as many as 100. Teleco wants to find a shipping and production plan that will maximize profit.

From	To (Rs '00)		
	Customer 1	Customer 2	Customer 3
Plant 1	75	60	69
Plant 2	79	73	68
Plant 3	85	76	70

i. Formulate balanced transportation problem that can be used to maximize Teleco's profit. (6 marks)

ii. Use northwest corner method to find a basic feasible solution to the problem. (6 marks)

iii. Use transportation simplex method to find an optimal solution to the problem. (6 marks)

b) Five workers are available to perform four jobs. The time it takes each worker to perform each job is given in the below table. The goal is to assign workers to jobs so as to minimize the total time required to perform the four jobs. Use Hungarian method to solve the problem.

Worker	Time (Hours)			
	Job 1	Job 2	Job 3	Job 4
1	10	15	10	15
2	12	8	20	16
3	12	9	12	18
4	6	12	15	18
5	16	12	8	12

(18 marks)

3) What is the major difference between trapezoidal and Simpson rules.

(2 mark)

Point out the advantage of having Simpson rule rather than trapezoidal rule.

(1 mark)

Simpson rule and trapezoidal rule can be used to approximate irregular integrals, but the error involving thies methods are significantly different, one student uses below regular integral to examine the error.

$$\text{If } f(x) = \int_0^4 \frac{1}{1+x^2} dx \ ,$$

a) Evaluate $f(x)$. (2 marks)

b) Approximate $f(x)$ by trapezoidal rule for 3 iterations and find the error.

(8 marks)

c) Approximate $f(x)$ by simpson rule for 3 iterations and find the error.

(8 marks)

d) Compare errors in part b and c.

(4 marks)

4) Explain the advantage of having cubic spline rather linear spline.

(2 marks)

a) In an experiment, following set of data is recorded.

$\{(1,3), (2,1), (3,2), (4,3)\}$

i. Find the piecewise linear interpolation function for the set of data.

(6 marks)

ii. Find the natural cubic spline that interpolates the data.

(8 marks)