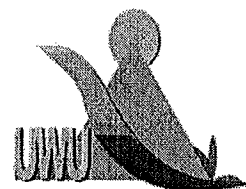




Uva Wellassa University, Sri Lanka
End Semester Examination – February / March 2012
SCT 301-2 Advanced Mathematics I



Time: Two (02) hours

Total Four (04) questions.

Answer all questions.

01. Define a linear function.

a) Show that the following functions are linear:

i. $f(x) = 2x$

ii. $f(u(t)) = \int_0^t u(\tau) d\tau$; where u is an integrable function.

iii. $f(t) = u(t+2)$; where u is a linear function

b) Define a time invariant system

Determine whether each of the following is time invariant or not, if it is given that $u(t)$ is time invariant.

i. $y(t) = 2u(t)$

ii. $y(t) = t u(t)$

02. a) Define the Laplace transform of a function $f(t)$ and show that the Laplace transform operator L is linear.

b) Find the Laplace transform of each of the following functions using the definition:

i. $f(t) = t^n ; n = 1, 2, 3, \dots$

ii. $f(t) = \sin \omega t$

iii. $f(t) = 5 \sin 2t - 3e^{3t}$

iv. $f(t) = \begin{cases} 3; & 0 < t < 2, \\ -1; & 2 < t < 4, \\ 0; & t \geq 4. \end{cases}$

03. a) Find each of the following inverse Laplace transformations

i. $L^{-1} \left\{ \frac{2}{s-3} \right\}$

ii. $L^{-1} \left\{ \frac{2s-8}{s^2+36} \right\}$

iii. $L^{-1} \left\{ \frac{s^3-s^2+s-1}{s^5} \right\}$

iv. $L^{-1} \left\{ \frac{s}{(s+3)(s+5)} \right\}$

b) Solve the following differential equations using Laplace transformation:

i. $y'''(t) + 8y(t) = 32t^3 - 16t, y(0) = y'(0) = y''(0) = 0$

ii. $f''(t) + 3f'(t) + 2f(t) = e^{-t}, f(0) = f'(0) = 0$

04. a) Define the following terms and give one example for each type:

i. Even function

ii. Odd function

iii. A periodic function with period T

b) Prove that $\int_{-L}^L \sin \frac{k\pi x}{L} dx = \int_{-L}^L \cos \frac{k\pi x}{L} dx = 0$ for $k = 1, 2, \dots$

c) Find the Fourier coefficients corresponding to the function

$$f(x) = \begin{cases} \cos x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$$