Uva Wellassa University of Sri Lanka
Faculty of Science and Technology
Department of Computer Science and Technology
100 level 1st Semester Examination – Dec. / Jan. 2018/19
CST 111-2 Discrete Mathematics I

Uva Wellassa University

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Instructions to candidates
Duration: Two (02) hours
Number of questions: Four (04) Essay Questions
Mark allocation: 100 mark
Use standard symbols without definition.
Scientific calculators are allowed.
Answer all questions

1.

a. Use a numerical table to estimate the limit of $f(x)=\dfrac{\sqrt{x+1}-3}{x-8}$ as x approaches to eight. (04 mark)

b. Evaluate the following limits.

i. $\lim\limits_{h\to 1}\dfrac{(h-1)^2}{(h^4-1)}$ (03 mark)

ii. $\lim\limits_{x\to 1}\left(\dfrac{\sqrt{1+2x}-\sqrt{3x}}{\sqrt{3+x}-2\sqrt{x}}\right)$ (03 mark)

iii. $\lim\limits_{t\to 3}\left(\dfrac{t^2}{t-3}-\dfrac{9}{t-3}\right)$ (03 mark)

iv. $\lim\limits_{x\to \infty}\left(\dfrac{2x^{2019}+3x^{2017}-6x^{1988}+3}{x^{2019}-5x^{2016}-x^{1991}-4}\right)$ (03 mark)

c. Obtain the first four terms of the following sequences.

i. $\left(\sqrt[n]{n}\right), n\in\mathbb{N}$ (02 mark)

ii. $\left(\dfrac{2^n+(-1)^n}{2^{n+1}+(-1)^{n+1}}\right), n\in\mathbb{N}$ (02 mark)

25

d. Determine whether the following sequences are *convergent* or *divergent*. Justify your answer.

i. $(12),\ n \in \mathbb{N}$ (02 mark)

ii. $\left(\frac{3n^2+8n+3}{n^2+5}\right),\ n \in \mathbb{N}$ (03 mark)

iii. $\left(1+(-1)^n\right),\ n \in \mathbb{N}$ (03 mark)

2.

a. Find the derivative of following functions with respective x.

i. $y = x^5 + 9x - x^{3/2} + \sqrt{x}$ (03 mark)

ii. $y = e^{e^{(x^3+3x)}}$ (03 mark)

iii. $y = \frac{\ln(x)}{\sqrt{x}}$ (03 mark)

iv. $y = (2x-1)\sqrt{x^2+2}$ (03 mark)

v. $y = \sqrt{x^2 - \frac{1}{x^2}}$ (03 mark)

b. If $y = \log_{10} x$, find $\frac{dy}{dx}$. (03 mark)

c. Let $f(x) = \frac{x^2}{x+1}$, then find $f'(x)$, $f''(x)$and $f'''(x)$. (04 mark)

3.

a. Evaluate following integrals with repective to x.

i. $\int\left(x^2+\frac{3}{\sqrt{x}}\right)dx$ (02 mark)

ii. $\int\left(3\sin x-5e^x+\cos ec^2 x\right)dx$ (02 mark)

iii. $\int_1^2\left(\sqrt{5x-1}\right)dx$ (03 mark)

iv. $\int_0^{\frac{\pi}{3}}\left(\sin x+\sec^2 x\right)dx$ (03 mark)

b. Evaluate the integral $\int\frac{1}{x(\ln x-2)}dx$ by using the **substitution method.**

(03 mark)

c. Use **Integration by parts** to show, $\int\frac{\ln t}{t^5}dt=-\frac{\ln t}{4t^4}-\frac{1}{16t^4}+C$, where C is an arbitrary constant.

(03 mark)

d. Find the values of constants A, B and D, such that $\frac{x^3-x^2+4x}{(x-1)(x^2+1)}=1+\frac{A}{(x-1)}+\frac{Bx+D}{(x^2+1)}$.

Hence, compute the integral $\int\frac{x^3-x^2+4x}{(x-1)(x^2+1)}dx$.

(*Hint:* $\int\frac{1}{a^2+x^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$, where a is a constant)

(04 mark)

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4.

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & -1 \\ 3 & -2 & -1 \end{bmatrix}$,

a. Find all minors of matrix A. (05 mark)

b. Find Cofactor matrix of A. (05 mark)

c. Find Adjoint matrix of A. (05 mark)

d. Find determinent of A. (05 mark)

e. If $A^{-1} = \frac{AdjA}{|A|}$, find Inverse matrix of A. (05 mark)

f. Hence, solve following system of equations.

$$x + 2y + z = 3$$

$$2x + 2y - z = -4$$

$$3x - 2y - z = 5$$

(05 mark)