Uva Wellassa University, Sri Lanka

B.Tech Degree Programme 2006/07

BSc in Computer Science Degree Programme 2006/07

End Semester Examination - Semester 1

January 2008

Math 201-2 Calculus

Instructions
Answer for **Four (04)** questions **only**
No. of questions: Five (05)
Time: Two hours
Formula sheets are provided at the end of this question paper

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(1)
(a) Suppose that **C** is a false statement, and **D** is a true statement
What is the truth-value of the compound statement **(~C)∨D**?

(b) Suppose that the compound statement **C⇒D** is a true statement.
In order for **C** to be true, what *must* the truth-value of **D** be?

(c) Suppose A and B are two statements. Construct a compound statement (containing both A and B) which is always false

(d) Give examples of two **infinite** sets A and B such that $A \subset B$

(e) Suppose $C \subseteq A$ and $C \subseteq B$. Prove that $C \subseteq A \cap B$

(2)
(a) What are the domains and ranges of the following functions?

(i) $f(x) = \dfrac{1}{\sqrt{9-x^2}}$ (ii) $f(x) = \tan(2x - \pi)$

(b) Find the following limits.

(i) $\lim\limits_{x \to 3} \dfrac{(3x+5)}{(4x-7)}$ (ii) $\lim\limits_{x \to a} \dfrac{(x^2 - a^2)}{(x-a)}$ (iii) $\lim\limits_{x \to 1} \dfrac{(1-\sqrt{x})}{(1-x)}$

(c) At what points is the following function continuous?

(i) $y = \frac{1}{(x-2)} - 3x$

(ii) For what value of a is the function

$$f(x) = \begin{cases} x^2 - 1 & , x < 3 \\ 2ax & x \geq 3 \end{cases}$$ continuous at every x? Explain.

(3)

(a) Differentiate following functions

(i) $y = \left(\frac{\sqrt{x}}{1+x}\right)^2$ with respect to x

(ii) $r = \left(\frac{\sin\theta}{\cos\theta - 1}\right)^2$ with respect to θ

(iii) $y = 2\sqrt{x}\sin\sqrt{x}$ with respect to x

(b)

(i) Graph the function

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & 1 < x \leq 2 \end{cases}$$

(ii) Is f continuous at $x = 1$?

(iii) Is f differentiable at $x = 1$?

Give reasons to your answers

(4)

(a) Evaluate following integrals

(i) $\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^3} dx$

(ii) $\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta}\cos^2\sqrt{\theta}} d\theta$

(iii) $\int_0^{\frac{\pi}{6}} \sqrt{1+\cos 6\theta}\, d\theta$

(iv) $\int_0^1 x\, e^x\, dx$

(b) Specific heat C_V is the amount of heat required to raise the temperature of a given gas with constant volume by 1° C, measured in units of cal/deg-mole (calories per degree gram molecule). The specific heat of oxygen depends on its temperature T and satisfies the formula

$$C_V = 8.27 + 10^{-5}\left(26T - 1.87T^2\right)$$

Find the average value of C_V for $20^{\circ} \leq T \leq 675^{\circ} C$ and the temperature at which it is attended. (Hint: use Mean Value Theorem)

(5)

(i) Find domains and ranges of following functions

(a) $f(x,y) = \sqrt{x^2 - y^2}$ (b) $f(x,y) = \dfrac{y^2}{\sin x}$ where x, y are independent variables.

(ii) Find following two limits

(a) $\displaystyle\lim_{(x,y)\to(0,-1)} \frac{y\cos x}{(xy+1)}$ (b) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{(x^2y^2-1)}{(xy-1)}$

(iii) $P(n,T,V) = \dfrac{nRT}{V}$ (the ideal gas law) where R is a constant

Find the partial derivative of P with respect to each variable.

(iv) Using the derivatives calculated in (iii), Find the change dP of the function $P(n,T,V) = \dfrac{nRT}{V}$ due to the changes dn, dT and dV.

FORMULA SHEET

Derivatives

(i) $\frac{d(k)}{dx} = 0$, *where k is a real constant.*

(ii) $\frac{d}{dx}(x^n) = nx^{n-1}$, *for each real number x and natural number n.*

(iii) $\frac{d}{dx}(\sin x) = \cos x$, *for all real numbers (radian measure) x.*

(iv) $\frac{d}{dx}(\cos x) = -\sin x$, *for all real numbers (radian measure) x.*

(v) $\frac{d}{dx}(\tan x) = \sec^2 x$, *for all real numbers* $x \neq (2n+1)\frac{\pi}{2}$, $n =$ *integer.*

(vi) $\frac{d}{dx}(\cot x) = -\csc^2 x$, *for all real numbers* $x \neq n\pi$, $n =$ *integer.*

(vii) $\frac{d}{dx}(\sec x) = \sec x \tan x$, *for all real numbers* $x \neq (2n+1)\frac{\pi}{2}$, $n =$ *integer.*

(viii) $\frac{d}{dx}(\csc x) = -\csc x \cot x$, *for all real numbers* $x \neq n\pi$, $n =$ *integer.*

Derivatives of Inverse functions

(i) $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$, $-1 < x < 1$.

(ii) $\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$, $-1 < x < 1$.

(iii) $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$, $-\infty < x < \infty$.

(iv) $\frac{d}{dx}(\text{arccot } x) = \frac{-1}{1+x^2}$, $-\infty < x < \infty$.

(v) $\frac{d}{dx}(\text{arcsec } x) = \frac{1}{|x|\sqrt{x^2-1}}$, $-\infty < x < -1$ *or* $1 < x < \infty$.

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TABLE Antiderivative formulas

	Function	General antiderivative
1.	x^n	$\frac{x^{n+1}}{n+1} + C,\quad n \neq -1,\ n$ rational
2.	$\sin kx$	$-\frac{\cos kx}{k} + C,\quad k$ a constant, $k \neq 0$
3.	$\cos kx$	$\frac{\sin kx}{k} + C,\quad k$ a constant, $k \neq 0$
4.	$\sec^2 x$	$\tan x + C$
5.	$\csc^2 x$	$-\cot x + C$
6.	$\sec x \tan x$	$\sec x + C$
7.	$\csc x \cot x$	$-\csc x + C$

Basic integration formulas

1. $\int du = u + C$
2. $\int k\,du = ku + C$ (any number k)
3. $\int (du + dv) = \int du + \int dv$
4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1)$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \sec^2 u\,du = \tan u + C$
9. $\int \csc^2 u\,du = -\cot u + C$
10. $\int \sec u \tan u\,du = \sec u + C$
11. $\int \csc u \cot u\,du = -\csc u + C$
12. $\int \tan u\,du = -\ln|\cos u| + C = \ln|\sec u| + C$
13. $\int \cot u\,du = \ln|\sin u| + C = -\ln|\csc u| + C$
14. $\int e^u\,du = e^u + C$
15. $\int a^u\,du = \frac{a^u}{\ln a} + C \quad (a > 0, a \neq 1)$
16. $\int \sinh u\,du = \cosh u + C$
17. $\int \cosh u\,du = \sinh u + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$
21. $\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C \quad (a > 0)$
22. $\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C \quad (u > a > 0)$