



Uva Wellassa University, Sri Lanka
End Semester Examination – March 2010
SCT 103-0 Basic Calculus



Time: Two (02) hours

Total 04 Questions
Answer all questions

1. (a) Determine whether each of the followings is function:

i. $y = x^2 + 1$ ii. $y^2 = x + 1$

[4 marks]

(b) Given $f(x) = \frac{x-1}{x+1}$; $x \neq -1$, find each of the followings:

i. $f(0)$ ii. $f\left(\frac{1}{2}\right)$ iii. $f(3x + 1)$ iv. $f\left(\frac{1}{x}\right)$ v. $f(2^x)$

[5 marks]

(c) If $f(x) = x^2$ and $g(x) = x^2 + 1$, find:

i. $g(g(0))$ ii. $f(g(2))$ iii. $f(g(f(1)))$ iv. $g(f(\sin x))$
v. $f(g(\cos x))$

[5 marks]

(d) Define the terms *domain* and *range* of a function.

Find the domain and the range of the following functions. Use interval notation where it is applicable.

i. $f(x) = x^3$ ii. $g(x) = \sqrt{x+2}$ iii. $h(x) = \frac{|x|}{x}$
iv. $p(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

[10 marks]

2. (a) Find the following limits if they exist:

i. $\lim_{y \rightarrow 3} |y|$ ii. $\lim_{t \rightarrow 5} |t - 5|$ iii. $\lim_{u \rightarrow \frac{3}{2}} \frac{u^2 - 4}{u - 2}$; $u \neq 2$
iv. $\lim_{x \rightarrow 3} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$; $x \neq 2$ v. $\lim_{v \rightarrow 1} \frac{v^2 - 1}{v - 1}$; $v \neq 1$ vi. $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 5x}$; $x \neq \frac{n}{5}\pi, n \in \mathbb{Z}$

[12 marks]

[P.T.O.]

- (b) If an object with mass m kilograms is dropped from rest, one model for its speed v after t seconds, taking air resistance into account, is

$$v = \frac{mg}{c} (1 - e^{-ct/m})$$

where g is the acceleration due to gravity and c is a positive constant.

- Calculate $\lim_{t \rightarrow \infty} v$.
- What is the meaning of this limit?

[5 marks]

- (c) Determine whether the following functions are continuous at the given point c :

i. $f(x) = \frac{3}{x-2}$ at $c = 2$

ii. $g(t) = \begin{cases} \frac{1}{t+4}, & t \neq -4 \\ 0, & t = -4 \end{cases}$ at $c = -4$

iii. $h(x) = \sqrt{3x-1}$ at $c = \frac{1}{3}$

iv. $p(x) = |x-2|$ at $c = 2$

[8 marks]

3. (a) The equation of motion is given for a particle, where s is in meters and t is in seconds.

Find the velocity and acceleration as function of t :

i. $s = 2t^3 - 15t^2 + 36t + 2, t \geq 0$

ii. $s = \sin\left(\frac{\pi t}{6}\right) + \cos\left(\frac{\pi t}{6}\right), 0 \leq t \leq 2$

iii. $s = \frac{t}{1+t^2}, t \geq 0$

[12 marks]

- (b) The volume of a right circular cone is $V = \pi r^2 h/3$, where r is the radius of the base and h is the height.

- Find the rate of change of the volume with respect to the height if the radius is constant.
- Find the rate of change of the volume with respect to the radius if the height is constant.

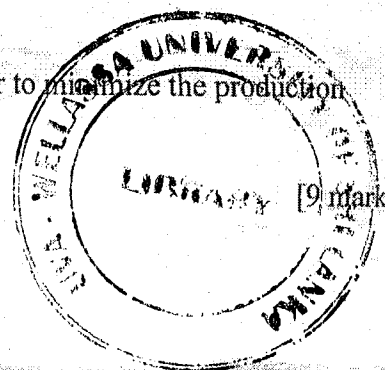
[4 marks]

- (c) A production facility is capable of producing 60,000 widgets in a day and the total daily cost of producing x widgets in a day is given by,

$$C(x) = 250,000 + 0.08x + \frac{200,000,000}{x}; x \neq 0$$

- What is the cost to produce the 552nd widget?
- What is the rate of change of the cost at $x = 500$?
- How many widgets per day should they produce in order to minimize the production cost?

[9 marks]



4. (a) Compute each of the following integrals:

i. $\int (8x^3 + 6x^2 - 2x + 3) dx$ ii. $\int (u - 1)(u - 2) du$

[4 marks]

(b) A particle moves along a line so that its velocity at time t is

$$v(t) = t^2 - t - 6 \text{ (measured in meters per second).}$$

Find the displacement of the particle during the time period $1 \leq t \leq 4$.

[Hint: $s(t_2) - s(t_1) = \int_{t_1}^{t_2} v(t) dt$; s is displacement]

[6 marks]

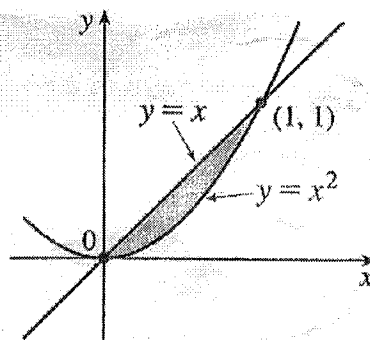
(c) The marginal cost $\left(\frac{dC(x)}{dx}$; $C(x)$ is cost) of manufacturing x yards of a certain fabric is

$$C'(x) = 3 - 0.01x + 0.000006x^2 \text{ (in rupees per yard).}$$

Find the increase in cost if the production level is raised from 2000 yards to 4000 yards.

[8 marks]

(d) Determine the area of the region enclosed by graphs $y = x^2$ and $y = x$.



[8 marks]