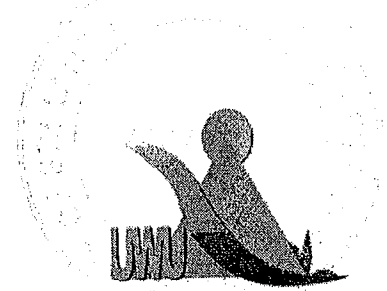


Uva Wellassa University, Sri Lanka.
End Semester Examination - June 2009
SCT 105-2 Mathematics I



Time : Two (2) hours

Answer all questions.

1. Point out the difference between an algebraic expression and an algebraic equation. (2 marks)
- a) An individual wants to use milk and orange juice to increase the amount of calcium and vitamin-A in her daily diet. An ounce of milk contains 30 mg of calcium and 50 mg of vitamin-A. An ounce of orange juice contains 5 mg of calcium and 60 mg of vitamin-A. How many ounce of milk and orange juice should he drinks each day to provide exactly 325 mg of calcium and 800 mg of vitamin A. (7 marks)
- b) Two pumps are used to fill a milk packet. One pump can fills the packet by itself in 9 seconds and the other can fills it in 6 seconds. How long it will takes if both pumps are operating together. (7 marks)
- c) How many liters of a mixture contains 80% of alcohol should be added to 5 liters of a 20% alcohol solution to yield a 30% solution. (8 marks)
2. What is the difference between series and sequences. (2 marks)
- a) If Rs1000 is invested at 6% annual interest, then after n years the total amount is a_n .
- Derive an expression for a_n .
 - Find the first three terms of the sequence $\{a_n\}$.
 - Is the sequence is convergent or divergent? Explain. (8 marks)

b) A certain ball has the property that each time it falls from a height h on to a hard level surface, it rebounds to a height rh , where $0 < r < 1$. Suppose that the ball is dropped from an initial height of H meters. (Use the fact that the ball falls $\frac{1}{2}gt^2$ meters in t seconds with g gravitational force)

- i. Assuming that the ball continues to bounce indefinitely, find the total distance it travels.
- ii. Calculate the total time that the ball travels.
- iii. Suppose that each time the ball strikes the surface with velocity v it rebounds with the velocity $-kv$, where $0 < k < 1$. How long will it take for the ball to come to rest.

(9 marks)

c) The size of an undisturbed fish population has been modeled by the formula;

$$p_{n+1} = \frac{bp_n}{a+p_n}$$

Where p_n is the fish population after n years and a and b are positive constants that depend on the species and its environment. Suppose that the population in year 0 is $p_0 > 0$.

- i. Show that if $\{p_n\}$ is convergent, then the only possible values for its limits are 0 and $b - a$.
- ii. Show that $p_{n+1} < (b/a)p_n$.
- iii. Use previous part to show that if $a > b$, then $\lim_{n \rightarrow \infty} p_n = 0$, and explain it.

(8 marks)

3. What is the major difference between an independent variable and a dependent variable.

(2 marks)

a) A manufacturer knows that it costs Rs9,000 to produce 1000 toaster ovens a week and Rs12,000 to produce 1500 toaster ovens a week.

- i. Express the cost as a function of the number of toaster ovens produced, assuming that it is linear. Then sketch the graph.
- ii. What is the slope of the graph and what does it represent.
- iii. What is the y-intercept of the graph and what does it represent.

(7 marks)

b) In a certain country, income tax is assessed as follows. There is no tax on income up to Rs10,000. Any income over Rs10,000 is taxed at a rate of 10% up to on income Rs20,000. Any income over Rs20,000 is taxed at 15%.

- i. Sketch the graph of the tax rate (R) as a function of the income (I).
- ii. How much tax is assessed on an income of Rs26,000.
- iii. Sketch the graph of the total assessed tax (T) as a function of income (I).

(7 marks)

c) A parking lot charges Rs50 for the first hour (or part of an hour) and Rs20 for each succeeding hour (or part) up to a daily maximum of Rs200.

- i. Sketch the graph of the cost of parking at this lot as a function of the time parked there.
- ii. Discuss the discontinuities of this function.

(7 marks)

4. Explain what is meant by partial differentiation.

(2 marks)

a) If R is the total resistance of three resistors, connected in parallel, with resistances R_1, R_2, R_3 , then $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$.

- i. If $A = \frac{1}{R}$, find $\frac{\partial A}{\partial R_1}$.
- ii. Find $\frac{\partial R}{\partial R_1}$ by using the above part.

(8 marks)

b) The temperature at a point (x, y) on a flat metal plate is given by $T(x, y) = \frac{60}{1+x^2+y^2}$, where T is measured in $^{\circ}\text{C}$ and x, y in meters. Find the rate of change of temperature with respect to distance at the point $(2, 1)$, in following directions.

- i. The x direction.
- ii. The y direction.

(8 marks)

c) The wind-chill index (W) is modeled by the function

$$W = 13.12 + 0.6215 T - 11.37 v^{0.16} + 0.3965 T v^{0.06}$$

Where T is the temperature ($^{\circ}\text{C}$) and v is the wind speed (km/h).

- i. Find rate of changing W with respect to T .
 - ii. Find rate of changing W with respect to v .
- when $T = 20^{\circ}\text{C}$ and $v = 30\text{km/h}$.

(8 marks)