



Uva Wellassa University, Sri Lanka  
End Semester Examination – March 2011  
SCT 103-0 Basic Calculus



Time: Two (02) hours

Total 04 Questions  
Answer all questions

1. (a) Determine whether each of the followings is function:

i.  $y = -11$

ii.  $y^2 = -x^2 + 1$

[4 marks]

(b) Given  $f(x) = \frac{x}{x+1}$ ;  $x \neq -1$ , find each of the followings:

i.  $f(0)$

ii.  $f\left(\frac{1}{2}\right)$

iii.  $f(3x + 1)$

iv.  $\frac{f(x+h)-f(x)}{h}$ ;  $h \neq 0$

[6 marks]

(c) If  $f(x) = x^2$  and  $g(x) = \sqrt{x^2 + 2}$ , find:

i.  $g(g(0))$

ii.  $f(g(2))$

iii.  $f(g(f(1)))$

iv.  $g(f(g(0)))$

[6 marks]

(d) Find the domain and the range of the following functions. Use interval notation where it is applicable.

i.  $f(x) = 5$

ii.  $g(x) = \sqrt{4 - x^2}$

iii.  $h(x) = |2x + 1|$

iv.  $p(x) = \frac{x}{3x-1}$

v.  $F(x) = \frac{1}{2}(x + 3)$

[10 marks]

2. (a) Find each of the following limits, if they exist:

i.  $\lim_{x \rightarrow 2} \frac{4x^3 + 5x + 7}{(x-4)(x+10)}$

ii.  $\lim_{t \rightarrow 2} \frac{t^2 - 4}{t - 2}$

iii.  $\lim_{u \rightarrow \infty} \frac{u^2 + u - 1}{2u + 5}$

iv.  $\lim_{x \rightarrow -\infty} \frac{3x^2 - 2x - 1}{x^3 + 4}$

v.  $\lim_{v \rightarrow 0} \frac{4v^3 - 2v^2 + v}{3v^2 + 2v}$

vi.  $\lim_{x \rightarrow -2} \sqrt{x^4 + 3x + 6}$

[12 marks]

(b) Determine whether the following functions are continuous at the given point  $c$ :

i.  $f(x) = \frac{3}{x-2}$  at  $c = 2$

ii.  $g(t) = \begin{cases} \frac{1}{t-1}, & \text{if } t \neq 1 \\ 2, & \text{if } t = 1 \end{cases}$  at  $c = 1$

iii.  $h(x) = \begin{cases} x + 3, & \text{if } x \neq 3 \\ 6, & \text{if } x = 3 \end{cases}$  at  $c = 3$

iv.  $p(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1}, & \text{if } x \neq 1 \\ 1, & \text{if } x = 1 \end{cases}$  at  $c = 1$

[12 marks]

3. (a) Differentiate each of the following functions with respect to  $x$ .

i.  $f(x) = x^3 + 3x - 5$

iv.  $q(x) = \frac{x+1}{3x^2}; x \neq 0$

ii.  $g(x) = [x^3 + 5]^3$

v.  $z(x) = [x^3 \sin x + \tan x - 2]$

iii.  $h(x) = [(x + 4)(x + 2)(x - 3)]$

[10 marks]

(b) A spherical balloon is inflated at a rate of  $100 \text{ ft}^3/\text{min}$ . Assuming the rate of inflation remains constant, how fast is the radius of the balloon increasing at the instant the radius is  $4 \text{ ft}$ ?

[6 marks]

(c) A small retailer can sell  $n$  units of a product for a revenue of  $r(n) = 8.1n$  and at a cost of  $c(n) = n^3 - 7n^2 + 18n$ , with all amounts in thousands rupees. How many units does it sell to maximize its profit?

[Hint: profit = revenue - cost]

[10 marks]

4. (a) Compute each of the following integrals:

i.  $\int (4x^3 + 3x^2 - 2x + 3) dx$

iii.  $\int_0^2 (6x^2 - 4x + 5) dx$

ii.  $\int \frac{4u^6 - u^4 + u^2 - 3}{u^3} du; u \neq 0$ .

iv.  $\int_{-1}^0 (2x - e^x) dx$

[10 marks]

(b) A force of  $1200 \text{ N}$  compresses a spring from its natural length of  $18 \text{ cm}$  to a length of  $16 \text{ cm}$ . How much work is done in compressing it from  $16 \text{ cm}$  to  $14 \text{ cm}$ ?

[Hint: The work done by the force along the  $x$ -axis from  $a$  to  $b$  is:  $W = \int_a^b kx dx$ , where  $k$  is the spring constant. Force  $F = kx$ ]

[6 marks]

(c) The work done when electric charges move toward each other (or when they are separated) is given by:  $W = \int_a^b \frac{kq_1q_2}{x^2} dx$ , where  $q_1$  and  $q_2$  are in coulombs (C),  $x$  is in metres, the force is in newtons and  $k$  is a constant,  $k = 9 \times 10^9$ .

An electron has a  $1.6 \times 10^{-19} \text{ C}$  negative charge. How much work is done in separating two electrons from  $1.0 \text{ pm}$  to  $4.0 \text{ pm}$ ?

[pm means *picometre*, or  $10^{-12}$  metres.]

[8 marks]