Uva Wellassa University, Sri Lanka
Faculty of Science and Technology
Science and Technology Degree program
2nd Semester Examination – September/October 2013

Uva Wellassa University

SCT 102-2 / SCT 105-2 Mathematics I

Number of questions: Four (04)
Answer **all** questions
Time allocation: **Two (02)** hours
Total marks allocated: 100

1.

a. Let a function $f(x)=x^2+1$ defined on $[-1,1]$.

i. Draw f on $[-1,1]$.

ii. State the domain and the range of f .

iii. For which values of x , $f(x)$ is 2 ?

iv. On which interval is f increasing?

v. On which interval is f decreasing.?

(10 marks)

b. Suppose that the height (in millimeters) of a small corn seedling after t days from germination is given by $h(t)=\dfrac{100}{3+5e^{-t}}$

i. What is the height of the corn seedling?
ii. What is the eventual height (the height when $t\to\infty$) of the plant?

(10 marks)

c. Find the constant c that makes f continuous on $(-\infty,\infty)$

$$f(x)=\begin{cases} x^2-c^2, \text{ if } x<4 \\ cx+20, \text{ if } x\geq 4 \end{cases}$$

(5 marks)

2.

a.

i. State the Intermediate Value Theorem and the Mean Value Theorem.

ii. If $f(x)=x^3-x^2+\ln(\frac{x}{2})$, show that there is a number c so that $f(c)=0$

iii. Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function.

$$f(x)=x^3+2x^2-x \quad ; \text{on} \quad [-1,2]$$

(15 marks)

b. If an initial amount A_0 of money is invested at an interest rate i compounded n times a year, the value of the investment after t years is $A=A_0(1+\frac{i}{n})^{nt}$. Show that $A=A_0e^{it}$ when interest is compounding continuously (i.e. let $n\to\infty$)

(10 marks)

3.

a. An amount Rs. 15000 is deposited in a bank account at 5% interest, compounded annually. Then the balance after **n** years is found by using the formula

$$a_n = 15000\left(1+\frac{5}{100}\right)^n$$

i. Find the account balance after six years.
ii. Find the first five terms of the sequence.
iii. State whether the sequence is increasing or decreasing.
iv. Is the sequence convergent or divergent?

(10 marks)

b.

i. Define sequences and series.
ii. What are the difference between a sequence and series.
iii. Under which conditions does an alternating series converge?
iv. Test the convergence of the series

$$\sum_{n=1}^{\infty}\left(\frac{5(n+1)}{5n+4}\right)^n$$

(15 marks)

4.

a. In a study of frost penetration it was found that the temperature T at time t at a depth x can be modeled by the function $T(x,t)=T_0+T_1e^{-\lambda x}\sin(\omega t-\lambda x)$. Where T_0, T_1, ω and λ are positive constants.

i. Find $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial t}$

ii. Show that T satisfies the heat equation $T_t = k\,T_{xx}$ for a certain constant k .

(10 marks)

b. Determine whether the following integrals convergent or divergent. If it is convergent find its valve.

i. $\int_{-2}^{2} \frac{1}{x^4} dx$

ii. $\int_{-2}^{2} \frac{1}{x^4+1} dx$

iii. $\int_{0}^{1} \frac{1}{x} dx$

iv. $\int_{1}^{\infty} \frac{1}{x} dx$

(15 marks)