Forecasting Foreign Exchange Rate using the Kalman Filter Approach

P.T. Mapatuna, R.R.L.U.I. Rajapaksha *and* J.K. Wijerathne
Department of Mathematics, Faculty of Science, University of Colombo

Introduction

The key element to fuel the 'price trend analysis' is the financial tick data (trade data). Tick data has several challenging features of which we intend to consider the most prominent setback when analyzing the price trend, which is data corruption and outliers. This will be our primary concern when building the model. There are different noises found in financial tick data namely process noise, measurement noise and arrival noise. These noises have been broadly studied in the engineering field for the case of an identified deterministic system. The Kalman filter was invented to approximate the state vector of a linear deterministic system in the presence of the process, measurement, and arrival noise. The Kalman filter has been applied in the field of econometrics for the case when a deterministic system is unknown and must be estimated from the data (Lumengo, 2008; Martinelli, 1995).

Many different methods have been offered to deal with signal mining problems in common and trend estimation has also received a great deal of attention especially when the interest is focused on forecasting turning points. In spite of all the differences among methods, one common feature remains in most of them. This is that trends tend to extrapolate themselves into the future as a line with a slope that depends on the recent past information. Although this is an optimal (eg. in a Mean Square Error sense) and a sensible way to progress, it can be systematically erroneous when turning points are at hand. Hence modeling the trend accurately was second principal concern.

The Kalman filter was incorporated in order to reduce the noise in the measurements and to obtain forecasted values of the exchange rates.

Methodology

Exchange rate for USD, GBP, EURO, CHF and YEN against rupee for the period from January 2005 to December 2010 was obtained from the Central Bank of Sri Lanka and was used for the trend analysis. The variables which cause significant impact to the Kalman filter model are the transition matrix and the process error. In order to obtain the most appropriate and optimal Kalman model five separate approaches, which indicated the most prominent effects, were used. Four out of the five methods were developed by altering the transition matrix that was incorporated in the model. The final method was obtained by letting the state variable be the trend between two consecutive days, here the transition characteristic of the tick data was included in the state variable itself, and hence, no transition matrix was utilized in the model.

Case 1: Taylor series method is used to derive the standard transition matrix

$F_t = e^{\Delta t A} = I + \sum_{i=1}^{\infty} \frac{\Delta t^i A^i}{i!} = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix}$. Here any power of A>1 yielded a zero matrix since

$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. Hence when $\Delta t = 1$, $F_t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Case 2: When $\Delta t = 0.00396825396825397$ $F_t = \begin{pmatrix} 1 & 0.00396825397 \\ 0 & 1 \end{pmatrix}$

Case 3: Application of the transition matrix with exponential smoothing factors.

Holt's two-parameter exponential smoothing model extends simple exponential smoothing to include a linear-trend component. Here α is the state smoothing constant for the data ($0< \alpha <1$), while β is the trend smoothing constant for the trend estimate ($0<\beta <1$). Hence $F_t = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$

Case 4: Application of the three by three transition matrix with volatility as the third variable.

The state variable follows a generalized Wiener process with a drift rate of α and a variance rate of b^2 if $\partial x = \alpha\ \partial t + b\ \partial z$. Then $\partial x = \alpha\ \partial t + b\ \varepsilon\sqrt{\partial t}$. Let $t = 1$ then the equation will reduce to $\partial x = \alpha + b\ \varepsilon$ where α and b is the gradient and the volatility terms of the data set respectively.This gives rise to the following equation $x_{t+1} - x_t = \alpha + b\varepsilon \rightarrow x_{t+1} = x_t + \alpha + b\varepsilon$, and $\alpha_{t+1} = \alpha_t$, $b_{t+1} = b_t$. Hence $F_t = \begin{pmatrix} 1 & 1 & \varepsilon_t \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Case 5: Application of the trend between two consecutive days as the state variable.

Here a transition matrix was not identified due to the fact that the state considered in this case incorporates the transition characteristic of the tick data. The objective here is to estimate the trend and use it to predict future data values. To do this, the Kalman filter employs an auxiliary equation to predict a future trend value from previous trend values.

Results and discussion

For the purpose of identifying a buy or a sell signal: for each of the five cases a plot (Pi vs. time) was generated Table 1). The Kalman filter provided us with standard deviations of the predictions (by Q_t). Letting $\Delta y_t = x_{t|t-1} - y_{t-1}$ denote the predicted price change on day t, and σ_t the standard deviation of that prediction, an indicator for day t can be defined as $\pi_t = \frac{\Delta y_t}{\sigma_t}$. If Pi is positive then the predicted movement is upward, conversely, if Pi is negative, the predicted movement is downward.

As inferred from the results some models built yielded accurate results while the others did not. There were cases, for example Case 1 model fitted the USD exchange rates properly but it did not fit for Yen. This as we presumed resulted in due to the magnitude of the currency. Yen exchange rates are significantly smaller compared to the USD exchange rates – hence we assumed the model did not fit to Yen as it did for USD. Further research needs to be done in this area to identify a proper reason behind this picture.

Table 1: Mean Absolute Deviation for each currency

MAD	USD	GBP	EURO	CHF	YEN
Case 1	0.0408	0.656425	0.61818	0.566335	0.012658
Case 2	1.653542	3.075498	3.735229	3.888446	0.06173
Case 3	0.040493	0.659737	0.678844	4.407804	0.401212
Case 4	0.839383	20.78238	7.521565	0.521262	0.619706
Case 5	0.051803	0.006029	0.314781	0.017187	4.78E-07

Based on the results the implementation described here is remarkably effective in some cases i.e. Case 1: Dollar – rupee exchange rates. The effectiveness of the filter is traced to two primary factors: the nature of the data, and the efficiency of the filter, which is determined by the filter's model in combination with a particular data set. The results clearly indicate that the application of this scheme to a shorter time period, say the previous quarter or two, should yield the best values for tomorrow's prediction.

There is the concept that the price movement in financial markets are a random procedure and that it is completely changeable (per. communication). This is up to a degree relatively true for the reason that in realism assessing the particular development is not feasible. Clearly if it were to approximate the perfect tendency then every individual in the market will have admission to this material which will undoubtedly guarantee an opposing drive in the markets, there by challenging the accurate estimation. But it can be seen from the simulations used above that there is some likelihood of knowing the upcoming trend, though it is not 100% accurate, it still provides an adequate logical control for the Forex dealers to deal on. Efficient market theory suggest that the market responds to all information available, hence being unable to draw a proper function to model the behavior of the Forex Market incorporates an innate error in all the models obtained.

Further at the later part of our study, we have tried to confer about a use of an indicator to identify buy / sell signals. Based on the generated Pi graphs we assume that the hypothetical cutoff (straight line above and below the oscillating point of the graph) gives an indication as to when buying or selling should occur. Since the objective of our study did not lie within this area we strongly believe that further study in to this could shed some light to the subject in concern.

Conclusions

The objective of this paper was to forecast the foreign exchange rate using the Kalman filter approach. It was noted that some of the models used, varied in results when applied to two different currency tickers. Hence we conclude that the effectiveness of the filter remained upon two main factors: the nature of the data in concern and the accurateness of the filter (this is subjective to the data set in concern).

References

Lumengo, B.B. 2008. Modeling the Rand-Dollar Future Spot Rates: The Kalman Filter Approach, Working Paper No: 98.

Martinelli, R. 1995. Market Data Prediction with an Adaptive Kalman Filter, Haiku Laboratories.