Uva Wellassa University

Faculty of Management

Degree of Bachelor of Business Management in Entrepreneurship and Management

THIRD YEAR FIRST SEMESTER EXAMINATION –FEBRUARY/MARCH 2012

EMG 334-2/EMG 333-2 –Actuarial Statistics - I

Part C: Essay Questions

Answer **all** questions.

Marks allocation Part C: 50 Marks

1. The probability mass function of the Poisson distribution with mean λ is given as

$$f_X(x) = P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad where\ \lambda > 0\ and\ x = 0, 1, 2, \ldots$$

The coefficient of skewness of a distribution is defined to be $\gamma_1 = {\mu_3}/{\sigma^3}$, where $\mu_r = E[(X - \mu)^r]$ and $\mu = E[X]$.

For the given Poisson distribution with mean λ, following results can be used without proving.

$$E[X] = \lambda$$
$$E[X^2] = \lambda + \lambda^2$$
$$E[X^3] = \lambda^3 + 3\lambda^2 + \lambda$$

Show that $\gamma_1 = \frac{1}{\sqrt{\lambda}}$ for the Poisson distribution.

(10 Marks)

2. A certain insurance policy was introduced to the market by ABC life in year 2011. It was found that the number of claims ,Y, has the probability density function $f_Y(y)$ given by

y	1	2	3	4	5	6
$f_Y(y)$	2k	3k	4k	9k	10k	12k

a) If $f_Y(y)$ is indeed a probability density function, then determine the value of k.

b) Find the cumulative distribution function (c.d.f.) $F_Y(y)$ of Y, and sketch the graph of the c.d.f.

c) Find $P(2 \leq Y < 5)$.

(10 Marks)

3. The size of a claim X, which arises under a certain type of insurance contract, is to be modeled using an exponential random variable with positive parameter **θ** such that the probability density function is given by

$$f_X(x) = \theta e^{-\theta X} \quad where\ x > 0, and\ \theta > 0$$

a) Verify that the cumulative distributions function of $f_X(x)$ is $F_X(x) = 1 - e^{-\theta X}$ when $x > 0$.

b) Using CDF of X, show that

i. The median size of claim is, $med_X = \theta^{-1}\ ln\, 2$.

ii. $Q_1 = \theta^{-1}\ ln\frac{4}{3}$.

iii. $Q_3 = \theta^{-1}\ ln\, 4$.

iv. $IQR = \theta^{-1} ln 3$.

c) Show that the moment generating function is given by $M_X(t) = \left(\frac{\theta}{\theta - t}\right)\ for\ t < \theta$.

d) Using the MGF of X, verify that

i. The mean of the claim amounts is $E[X] = {}^1\!/_\theta$

ii. The variance of claim amounts of the company is $V[X] = {}^1\!/_{\theta^2}$

(30 Marks)