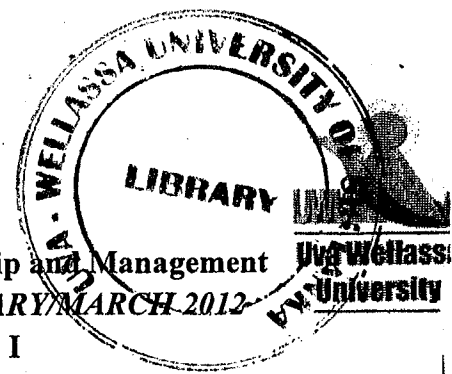


Uva Wellassa University
Faculty of Management

Degree of Bachelor of Business Management in Entrepreneurship and Management

THIRD YEAR FIRST SEMESTER EXAMINATION - FEBRUARY/MARCH 2012

EMG 334-2/EMG 333-2 - Actuarial Statistics - I
Part C: Essay Questions



Answer **all** questions.

Marks allocation Part C: 50 Marks

1. The probability mass function of the Poisson distribution with mean λ is given as

$$f_X(x) = P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} \text{ where } \lambda > 0 \text{ and } x = 0, 1, 2, \dots$$

The coefficient of skewness of a distribution is defined to be $\gamma_1 = \frac{\mu_3}{\sigma^3}$, where $\mu_r = E[(X - \mu)^r]$ and $\mu = E[X]$.

For the given Poisson distribution with mean λ , following results can be used without proving.

$$E[X] = \lambda$$

$$E[X^2] = \lambda + \lambda^2$$

$$E[X^3] = \lambda^3 + 3\lambda^2 + \lambda$$

Show that $\gamma_1 = \frac{1}{\sqrt{\lambda}}$ for the Poisson distribution.

(10 Marks)

2. A certain insurance policy was introduced to the market by ABC life in year 2011. It was found that the number of claims, Y , has the probability density function $f_Y(y)$ given by

y	1	2	3	4	5	6
$f_Y(y)$	2k	3k	4k	9k	10k	12k

- a) If $f_Y(y)$ is indeed a probability density function, then determine the value of k .
- b) Find the cumulative distribution function (c.d.f.) $F_Y(y)$ of Y , and sketch the graph of the c.d.f.
- c) Find $P(2 \leq Y < 5)$.

(10 Marks)

3. The size of a claim X , which arises under a certain type of insurance contract, is to be modeled using an exponential random variable with positive parameter θ such that the probability density function is given by

$$f_X(x) = \theta e^{-\theta x} \quad \text{where } x > 0, \text{ and } \theta > 0$$

- a) Verify that the cumulative distributions function of $f_X(x)$ is $F_X(x) = 1 - e^{-\theta x}$ when $x > 0$.
- b) Using CDF of X , show that
- The median size of claim is, $med_X = \theta^{-1} \ln 2$.
 - $Q_1 = \theta^{-1} \ln \frac{4}{3}$.
 - $Q_3 = \theta^{-1} \ln 4$.
 - $IQR = \theta^{-1} \ln 3$.
- c) Show that the moment generating function is given by $M_X(t) = \left(\frac{\theta}{\theta-t}\right)$ for $t < \theta$.
- d) Using the MGF of X , verify that
- The mean of the claim amounts is $E[X] = 1/\theta$
 - The variance of claim amounts of the company is $V[X] = 1/\theta^2$

(30 Marks)